Hydrodynamic Added-Mass Identification from Resonance Tests

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The finite element method applied to the calculation of hydrodynamic added mass implies error and high computer cost. The proposed method aims at identifying the added mass by using the measurement of the modes of the structure, both "dry" and in contact with the fluid. The discrete model which expresses the dynamic behavior of the fluid structure system is obtained through an optimization procedure. The prediction of the influence of structural modification was obtained by applying the method to the case of a plate partially immersed in water.

Nomenclature

$[A]$	$= [D]^{1/2}$
$[B]$	= matrix defined in Eq. (26)
$[C]$	= damping matrix
$[D]$	= matrix defined in Eq. (29)
F	= generalized force vector
$[G]$	= matrix associated with gravity forces in Eq. (6)
$[H]$	= "stiffness" matrix of the fluid
$[I]$	= identity matrix
$[K]$	= stiffness matrix of the structure
$[L]$	= interaction matrix used in Eq. (2)
m	= lumped mass used in the structural modification
$[m]$	= matrix defined in Eq. (23)
$[M]$	= modal added-mass matrix used in Eq. (6)
$[\bar{M}]$	= mass matrix of the structure
$[\tilde{M}]$	= added-mass matrix
n	= number of elastic modes used in Eq. (3)
p	= excitation point
q	= modal coordinate vector
Q	= force vector resulting from the fluid
$[R]$	= full modal damping matrix
T	= mode shapes of the immersed structure
$\tilde{T}$	= measured mode shapes of the immersed structure
X	= mode shapes of the dry structure
Y	= modal coordinate vectors of modes of the immersed structure
$\overset{*}{Y}$	= modal coordinate vectors of modes of the immersed structure with mass modification
$[Z]$	= intermediate matrix used in Eq. (17)
δ	= vector of nodal displacements
δ_{ij}	= Kronecker delta
$[\Delta M]$	= mass modification matrix defined in Eq. (41)
$[\lambda]$	= matrix of Lagrange multipliers
ρ	= mass density of the fluid
ϕ	= weighted Euclidean norm defined in Eq. (30)
Ψ	= Lagrange function defined in Eq. (32)
Ω	= angular natural frequencies of the "dry" structure
$\tilde{\Omega}$	= angular natural frequencies of the structure in contact with the fluid
$\overset{*}{\Omega}$	= angular natural frequencies of the immersed structure with mass modification

Superscripts

T	= matrix transpose
-1	= matrix inverse

Subscripts

E	= elastic mode
i,j,k,ℓ	= indices
p	= value at the excitation point p
R	= rigid mode

Introduction

THE hydrodynamic added mass can be calculated from the application of the finite element method to the fluid structure coupling.[1-3] As a rule, the fluid's incompressibility is taken for granted and the losses due to radiation and surface waves are discounted. Under these conditions the coupled problem can be written in the following simple form:

$$[K]\delta + [C]\dot{\delta} + [[\bar{M}] + [\tilde{M}]]\ddot{\delta} = 0 \tag{1}$$

The matrices $[K]$, $[C]$, and $[\bar{M}]$ represent the stiffness, damping, and mass of the structure, respectively. In general, they are obtained by the usual finite element methods.[4] The vector δ is composed of nodal displacements of the structure. The added-mass matrix is given by

$$[\tilde{M}] = \rho [L][H]^{-1}[L^T] \tag{2}$$

where ρ is the mass density of the fluid, $[L]$ the interaction matrix, and $[H]$ the "stiffness" matrix of the fluid. These matrices can be calculated by using the cubic fluid finite elements given in Ref. 3. Their application to the case of plates immersed in water gives results close to the experimental values.[5-7] A simplified method which takes the fluid's compressibility into account has recently been proposed.[8]

Using the finite element method automatically involves both errors in the calculation of the added mass and high computer costs. It is thus worthwhile to correct the mass matrix using an incomplete set of measured modes of the fluid structure system. The total mass matrix $[\bar{M}+\tilde{M}]$ can be corrected by imposing the measured modes orthogonality condition, using the method proposed by Bermann.[9] The mass matrix $[\bar{M}]$ can be corrected in the same way by using the modes of the "dry" structure. The method proposed by Baruch[10-12] can also be used to correct the added-mass matrix $[\tilde{M}]$. The procedure is based on the reorthogonalization of the modes with respect to the stiffness matrix $[K]$. With the help of the method given in Ref. 13, the modes are then used to obtain a more accurate estimation of the mass matrix $[\tilde{M}]$. All of these corrective techniques require an initial added-mass matrix obtained by the finite element method. This computation is always difficult. Thus it seems helpful to obtain a discrete model of the fluid structure system directly from vibration tests.

The main goal of this study is to obtain a model experimentally which can be used in the context of modal synthesis methods.[14] In the first section, a method of identification of the added mass in a set of measured normal modes of the dry structure is presented. The mass coupling terms are directly related to the change in shape of these modes when the structure is put in contact with the fluid. In the second section, the experimental model is corrected by an optimization procedure. In the final section, the proposed method is applied to the case of a free plate partially immersed in water. A structural modification, induced by the

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adding of mass to the nonimmersed part, enabled us to test the validity of the model.

Method Exposition

In accordance with standard modal synthesis methods using free modes,[15,16] the structure's displacement is approximated by the first normal modes obtained without fluid,

$$\delta = [X_R] q_R + [X_E] q_E \tag{3}$$

where $[X_R]$ is the modal matrix for r rigid body modes which may exist and $[X_E]$ the modal matrix of n first elastic modes. These normal modes satisfy the following orthogonality relationship:

$$[X_R X_E]^T [\bar{M}] [X_R X_E] = [I] \tag{4}$$

In Eq. (3) q_R and q_E are the modal coordinate vectors. When the structure is in contact with the fluid, these generalized coordinates satisfy the matrix equation,

$$\begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} \begin{Bmatrix} \ddot{q}_R \\ \ddot{q}_E \end{Bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & R \end{bmatrix} \begin{Bmatrix} \dot{q}_R \\ \dot{q}_E \end{Bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \Omega^2 \end{bmatrix} \begin{Bmatrix} q_R \\ q_E \end{Bmatrix} = \begin{Bmatrix} F_R \\ F_E \end{Bmatrix} + \begin{Bmatrix} Q_R \\ Q_E \end{Bmatrix} \tag{5}$$

The spectral matrix $[\Omega^2]$ is constituted by the square of the angular frequencies of the modes obtained without fluid, and $[R]$ is the full modal damping matrix associated with the n elastic modes. The modal force vectors Q_R and Q_E resulting from the fluid can be reduced to a simple matrix form,

$$\begin{Bmatrix} Q_R \\ Q_E \end{Bmatrix} = - \begin{bmatrix} G & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} q_R \\ q_E \end{Bmatrix} - \begin{bmatrix} M_{RR} & M_{RE} \\ M_{ER} & M_{EE} \end{bmatrix} \begin{Bmatrix} \ddot{q}_R \\ \ddot{q}_E \end{Bmatrix} \tag{6}$$

In this equation the fluid is assumed to be incompressible. The matrix $[G]$ corresponds to the gravity forces which are assumed to intervene only in rigid body displacements. The modal force vectors F_R and F_E are additional generalized forces which do not result from the fluid. The matrix $[M_{RR}]$ corresponds to the added mass associated with the rigid body modes. The matrix $[M_{EE}]$ is the hydrodynamic added-mass matrix and $[M_{ER}]$ is the coupling added-mass matrix. The first modes of the fluid structure system may be approximated from the model defined in Eqs. (5) and (6). The discrete normal modes are solutions of the following eigenvalue problem:

$$\begin{bmatrix} G & 0 \\ 0 & \Omega^2 \end{bmatrix} \begin{Bmatrix} Y_R \\ Y_E \end{Bmatrix}_i = \bar{\Omega}_i^2 \begin{bmatrix} I+M_{RR} & M_{RE} \\ M_{ER} & I+M_{EE} \end{bmatrix} \begin{Bmatrix} Y_R \\ Y_E \end{Bmatrix}_i \tag{7}$$

The modal coordinates of each eigenvector give an approximate mode shape T_i of the immersed structure,

$$T_i = [X_R X_E] \begin{Bmatrix} Y_R \\ Y_E \end{Bmatrix}_i \tag{8}$$

In numerous cases, the r first natural frequencies of the immersed structure are much lower than the n other natural frequencies. Under these conditions, it is advantageous to collect Eq. (7) in the following matrix form:

$$\begin{bmatrix} G & 0 \\ O & \Omega^2 \end{bmatrix} \begin{bmatrix} Y_{RR} & Y_{RE} \\ Y_{ER} & Y_{EE} \end{bmatrix} = \begin{bmatrix} I+M_{RR} & M_{RE} \\ M_{ER} & I+M_{EE} \end{bmatrix} \begin{bmatrix} Y_{RR} & Y_{RE} \\ Y_{ER} & Y_{EE} \end{bmatrix} \begin{bmatrix} \bar{\Omega}_R^2 & 0 \\ 0 & \bar{\Omega}_E^2 \end{bmatrix} \tag{9}$$

The spectral matrix $[\bar{\Omega}_R^2]$ is constituted by the square of the angular natural frequencies of the r first modes. The dimension of the spectral matrix $[\bar{\Omega}_E^2]$ is n. Equation (9) is thus broken down into four relationships,

$$[G][Y_{RR}] = [I+M_{RR}][Y_{RR}][\bar{\Omega}_R^2] + [M_{RE}][Y_{ER}][\bar{\Omega}_R^2] \tag{10}$$

$$[\Omega^2][Y_{ER}] = [M_{ER}][Y_{RR}][\bar{\Omega}_R^2] + [I+M_{EE}][Y_{ER}][\bar{\Omega}_R^2] \tag{11}$$

$$[G][Y_{RE}] = [I+M_{RR}][Y_{RE}][\bar{\Omega}_E^2] + [M_{RE}][Y_{EE}][\bar{\Omega}_E^2] \tag{12}$$

$$[\Omega^2][Y_{EE}] = [M_{ER}][Y_{RE}][\bar{\Omega}_E^2] + [I+M_{EE}][Y_{EE}][\bar{\Omega}_E^2] \tag{13}$$

If the r first natural frequencies of the immersed structure are much weaker than the natural frequencies of the flexural modes of the dry structure, Eq. (11) gives an approximate expression of the modal matrix $[Y_{ER}]$,

$$[Y_{ER}] = [\Omega^{-2}][M_{ER}][Y_{RR}][\bar{\Omega}_R^2] \tag{14}$$

Thus, with the low-mode assumption, the r first modes are close to rigid body modes. Under these conditions, Eq. (10) gives an approximate expression of the stiffness matrix $[G]$,

$$[G] = [I+M_{RR}][Y_{RR}][\bar{\Omega}_R^2][Y_{RR}]^{-1} \tag{15}$$

Introducing Eq. (15) into Eq. (12) we obtain,

$$[I+M_{RR}][Y_{RR}][-[\bar{\Omega}_R^2][Z] + [Z][\bar{\Omega}_E^2]] + [M_{RE}][Y_{EE}][\bar{\Omega}_E^2] = 0 \tag{16}$$

where

$$[Z] = [Y_{RR}]^{-1}[Y_{RE}] \tag{17}$$

If it is supposed that the r first natural frequencies of the immersed structure are much lower than its other natural frequencies, the matrix $[\bar{\Omega}_R^2][Z]$ may be neglected with respect to the matrix $[Z][\bar{\Omega}_E^2]$. Under these conditions, Eq. (16) gives an approximate expression of the coupling added-mass matrix,

$$[M_{RE}] = -[I+M_{RR}][Y_{RE}][Y_{EE}]^{-1} \tag{18}$$

The hydrodynamic added-mass matrix $[M_{EE}]$ can be calculated from Eq. (13),

$$[M_{EE}] = [\Omega^2][Y_{EE}][\bar{\Omega}_E^2]^{-1}[Y_{EE}]^{-1} - [I] - [M_{ER}][Y_{RE}][Y_{EE}]^{-1} \tag{19}$$

With the help of Eq. (18), this mass matrix can be written in the form,

$$[M_{EE}] = [\Omega^2][Y_{EE}][\bar{\Omega}_E^2]^{-1}[Y_{EE}]^{-1} - [I] + [Y_{EE}]^{-T}[Y_{RE}]^T[I+M_{RR}][Y_{RE}][Y_{EE}]^{-1} \tag{20}$$

The exact terms M_{ij} of the matrices $[M_{EE}+I]$ and $[M_{ER}]$ are given by

$$M_{ij} = T_i^T [\bar{M} + \tilde{M}] T_j \tag{21}$$

However, Eqs. (18) and (20) give an approximate value for these terms without calculating $[\tilde{M}]$. The modal coordinates of the n elastic modes of the structure in contact with the fluid can be calculated from the mode shapes T_i by using Eq. (8) and the orthogonality relationships [Eq. (4)] of the modes of the "dry" structure,

$$\begin{bmatrix} Y_{RE} \\ Y_{EE} \end{bmatrix} = \begin{bmatrix} X_R^T \\ X_E^T \end{bmatrix} [\bar{M}][T] \tag{22}$$

Thus, the added mass matrices $[M_{EE}]$ and $[M_{ER}]$ given in Eqs. (18) and (20) can be calculated by measuring the mode shapes T_i at the nodal points. In order to prove the orthogonality condition of Eq. (4) of the case in which the "dry" modes are obtained by experiment, the structure's mass matrix $[\bar{M}]$ can be corrected by Berman's method.[9] The added mass matrix $[M_{RR}]$ is identified independently by appropriate experimental methods, such as pendulum tests. The actual mode shapes $\tilde{T}_i$ obtained by experiment differ slightly from the approximate normal modes T_i defined in Eqs. (7) and (8). Thus, the first matrix occurring in the expression of $[M_{EE}]$, given in Eq. (20), is not precisely symmetrical. The vectors T_i and $\tilde{T}_i$ may differ for several reasons: 1) the influence of modal truncation in Eq. (3), 2) the influence of losses due to surface waves and radiation in the fluid, and 3) errors in the measurement of mode shapes. In order to obtain a symmetrical hydrodynamic added-mass matrix, the proposed method uses an optimization procedure to correct the measured modes.

Correction of the Model

Equation (20) shows that the following matrix must be symmetrical:

$$[m] = [\Omega^2][Y_{EE}][\overline{\Omega_E^2}]^{-1}[Y_{EE}]^{-1} \tag{23}$$

If the matrix $[m]$ is symmetrical, the same is true for the matrix $[Y_{EE}]^T[m][Y_{EE}]$. Using Eq. (23), we obtain

$$[Y_{EE}]^T[m][Y_{EE}] = [Y_{EE}]^T[\Omega^2][Y_{EE}][\overline{\Omega_E^2}]^{-1} \tag{24}$$

Thus, the symmetry condition of matrix $[m]$ is equivalent to the following simple equation:

$$[B][\overline{\Omega_E^2}]^{-1} = [\overline{\Omega_E^2}]^{-1}[B] \tag{25}$$

with

$$[B] = [Y_{EE}]^T[\Omega^2][Y_{EE}] \tag{26}$$

This equation is proved when, and only when, either $[B]$ is diagonal or all the elements of the spectral matrix $[\overline{\Omega_E^2}]$ are equal. But the latter alternative is not admissible since it implies that all the natural frequencies of the immersed structure are identical. Thus, Eq. (25) requires that the matrix $[B]$ be diagonal. This condition can be written in the following simple form:

$$[B] = [Y_{EE}]^T[\Omega^2][Y_{EE}] = [I] \tag{27}$$

By substituting $[Y_{EE}]$ for its expression given in Eq. (22), this equation becomes

$$[T]^T[D][T] = [I] \tag{28}$$

with

$$[D] = [\bar{M}][X_E][\Omega^2][X_E]^T[\bar{M}] \tag{29}$$

The mass matrix $[M_{EE}]$ is symmetrical if, and only if, the orthogonality relationship of Eq. (28) is satisfied. The mode shapes T_i used in the calculation of the added masses will be obtained by correcting the experimental mode shapes $\tilde{T}_i$ in order to prove Eq. (28). The corrected modes are obtained through the use of the optimization procedure proposed by Baruch.[10] The problem is to find a matrix $[T]$ that minimizes the weighted Euclidean norm

$$\phi = \| [A][[T]-[\tilde{T}]] \| = \sum_{i=1}^{m}\sum_{k=1}^{n}\left(\sum_{j=1}^{m} A_{ij}(t_{jk}-t_{\tilde{j}k})^2\right) \tag{30}$$

with

$$[A] = [D]^{1/2} \tag{31}$$

and satisfies the orthogonality condition given in Eq. (28). The following Lagrange function is defined by using Lagrange multipliers:

$$\Psi = \phi + \| [\lambda]([T]^T[D][T]-[I]) \| \tag{32}$$

where $[\lambda]$ is a symmetric matrix of Lagrange multipliers and

$$\| [\lambda]([T]^T[D][T]-[I]) \| = \sum_{i=1}^{n}\sum_{\ell=1}^{n}\lambda_{\ell i}\left(\sum_{j=1}^{m}\sum_{k=1}^{m}(t_{ji}d_{jk}t_{k\ell}-\delta_{\ell i})\right) \tag{33}$$

$\delta_{\ell i}$ is the Kronecker delta. The corrected matrix $[T]$ is a solution of the following matrix equation:

$$\frac{\partial\Psi}{\partial T} = 2[D][T-\tilde{T}] + 2[D][T][\lambda] = 0 \tag{34}$$

Since the proposed method brings in the rigid-body modes separately, matrix $[D]$ can be inverted. Thus, Eq. (34) gives

$$[T][I+\lambda] = [\tilde{T}] \tag{35}$$

Using the orthogonality condition of Eq. (28) and the procedure proposed by Baruch,[10] the final result obtained is

$$[T] = [\tilde{T}][[\tilde{T}][D][\tilde{T}]]^{-1/2} \tag{36}$$

The matrix $[T]$ can be obtained by using the iterative dual process as follows

$$[T_{k+1}] = \tfrac{1}{2}[T_k][[I] + [[T_k]^T[D][T_k]]^{-1}] \tag{37a}$$

with

$$[T_0] = [\tilde{T}] \tag{37b}$$

Once $[\tilde{T}]$ is known, the added-mass matrices $[M_{ER}]$ and $[M_{EE}]$ can be calculated. The entire procedure may be summarized as follows:

Step 0. Measure the n first modes of the dry structure and the n first modes of the immersed structure at the nodal points distributed over the surface of the structure. These two modal identifications can be carried out using one of the methods given in Refs. 17 and 18.

Step 1. Correct the mass matrix of the dry structure so that the measured modes satisfy Eq. (4) by using the method in Ref. 9.

Step 2. Calculate matrix $[D]$ defined in Eq. (29).

Step 3. Calculate the corrected modal matrix $[T]$ by using the iterative process given in Eq. (37).

Step 4. Compute the modal coordinate matrices $[Y_{RE}]$ and $[Y_{EE}]$ with the help of Eq. (22).

Step 5. Compute the hydrodynamic added-mass matrices $[M_{RE}]$ and $[M_{EE}]$ by using Eqs. (18) and (20).

The application of the method to a sample problem will allow us to test the validity of the model obtained.

Validity Test of the Method

The structure studied is a rectangular steel plate suspended by wires, see Fig. 1. The three rigid-body modes are constituted by one translation and two rotations associated with the boundary coordinate system q_R shown in Fig. 1. The flexural modes are measured at 24 points distributed over the surface of the plate. The six first normal modes are identified. Their natural frequencies are given in Table 1.

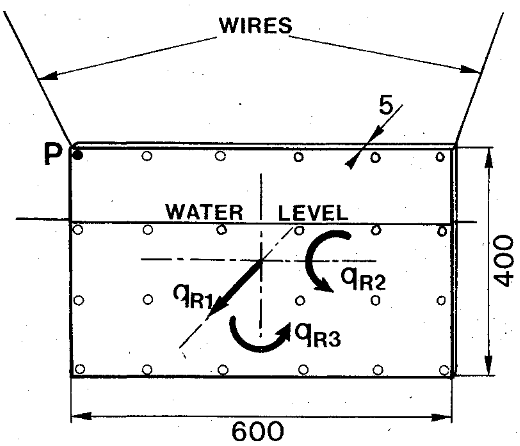

Fig. 1 Free rectangular plate immersed in water.

Table 1 Natural frequencies of the first six elastic modes

No. of modes	Measured natural frequencies, Hz: Dry structure	Immersed structure
1	75.7	48.6
2	84.6	64.9
3	169.5	118.4
4	178.5	132.6
5	205.6	167.9
6	238.7	141.6

The plate is immersed to two-thirds of its height in water (see Fig. 1). The matrix $[G]$ associated with gravity forces is zero. The mass matrix $[M_{RR}]$ is obtained by pendulum tests. The natural frequencies of the first six flexural modes measured when the structure is in contact with water are given in Table 1. The coordinates of these six measured modes in the set composed of the three rigid-body modes and the first six flexural dry modes are given in Table 2. The matrix $[\tilde{T}]^T[D][\tilde{T}]$ is close to the unitary matrix, see Table 3. The corrected modes $[T]$, proving the orthogonality condition [Eq. (28)], are given in Table 4. Their modal coordinates, which were calculated using Eq. (22), are close to those of the uncorrected modes as shown in Table 2. The coupling added-mass matrix calculated on the basis of Eq. (18) is

$$[M_{RE}] = \begin{bmatrix} 0.065 & 0 & -0.369 & 0.019 & 0 & 0 \\ 0.043 & 0 & -0.255 & 0.089 & 0 & 0 \\ 0 & -0.715 & 0 & 0 & -0.363 & -0.212 \end{bmatrix} \tag{38}$$

The hydrodynamic added-mass matrix $[M_{EE}]$ is obtained by using Eq. (20),

$$[M_{EE}] = \begin{bmatrix} 1.375 & 0 & 0.087 & -0.832 & 0 & 0 \\ & 0.962 & 0 & 0 & 0.094 & -0.348 \\ & & 1.176 & 0.058 & 0 & 0 \\ & & & 0.964 & 0 & 0 \\ & S & & & 1.472 & -0.062 \\ & & & & & 1.186 \end{bmatrix} \tag{39}$$

Table 2 Modal coordinates of the six first elastic modes of the fluid structure system

Mode No.	Rigid-body modes 1	2	3	Bending modes 4	5	6	7	8	9
1	0.130 E-1	0	0.368 E-3	-0.104 E-2	0	0	-0.447 E-3	-0.259 E-3	0
	0.130 E-1[a]	*0*	*0.134 E-3*	*-0.942 E-3*	*0*	*0*	*-0.507 E-3*	*-0.307 E-3*	*0*
2	0	0.117 E-1	0	0	-0.122 E-3	-0.606 E-3	0	0	0.477 E-2
	0	*0.118 E-1*	*0*	*0*	*-0.668 E-4*	*-0.444 E-3*	*0*	*0*	*0.452 E-2*
3	0.261 E-3	0	0.584 E-2	0.785 E-3	0	0	0.140 E-2	0.965 E-3	0
	0.129 E-3	*0*	*0.579 E-2*	*0.106 E-2*	*0*	*0*	*0.139 E-2*	*0.944 E-3*	*0*
4	0.204 E-2	0	-0.142 E-2	0.537 E-2	0	0	-0.503 E-3	-0.626 E-3	0
	0.224 E-2	*0*	*-0.111 E-2*	*0.542 E-2*	*0*	*0*	*-0.438 E-3*	*-0.532 E-3*	*0*
5	0	-0.263 E-4	0	0	0.482 E-2	-0.552 E-3	0	0	0.881 E-3
	0	*0.189 E-4*	*0*	*0*	*0.483 E-2*	*-0.476 E-3*	*0*	*0*	*0.909 E-3*
6	0	0.111 E-2	0	0	0.445 E-3	0.415 E-2	0	0	0.100 E-2
	0	*0.126 E-2*	*0*	*0*	*0.549 E-3*	*0.414 E-2*	*0*	*0*	*0.108 E-2*

[a] Corrected values are in italics.

Table 3 Modal stiffness matrix $[\tilde{T}]^T[D][\tilde{T}]$

1.000 E+0	0	0.550 E-1	-0.416 E-1	0	0
0	1.000 E+0	0	0	-0.798 E-2	-0.524 E-1
0.550 E-1	0	1.000 E+0	-0.101 E+0	0	0
-0.416 E-1	0	-0.101 E+0	1.000 E+0	0	0
0	-0.798 E-2	0	0	1.000 E+0	-0.401 E-1
0	-0.524 E-1	0	0	-0.401 E-1	1.000 E+0

Table 4 Corrected mode shapes of the immersed plate

Data Point	Rigid-body modes 1	2	3	Bending modes 1	2	3	4	5	6
1	0.366	0.574	0.610	0.638	0.845	0.456	0.857	0.778	0.446
2	0.366	0.574	0.366	0.142	0.621	0.536	0.271	0.039	0.642
3	0.366	0.574	0.122	−0.254	0.227	0.604	−0.482	−0.156	0.362
4	0.366	0.191	0.610	0.638	0.317	−0.275	0.430	0.389	−0.562
5	0.366	0.191	0.366	0.057	0.222	−0.226	0.026	−0.368	0.144
6	0.366	−0.191	0.122	−0.389	−0.081	−0.190	0.271	−0.347	0.056
7	0.366	−0.191	0.610	0.638	−0.317	−0.275	−0.430	0.389	−0.562
8	0.366	−0.191	0.366	0.057	−0.222	−0.226	−0.026	−0.368	−0.144
9	0.366	−0.191	0.122	−0.389	−0.081	−0.190	0.271	−0.347	0.056
10	0.366	−0.574	0.610	0.638	−0.845	0.456	−0.857	0.778	0.466
11	0.366	−0.574	0.366	0.149	−0.621	0.536	−0.271	0.039	0.642
12	0.366	−0.574	0.122	−0.254	−0.227	0.604	0.482	−0.233	0.362
13	0.366	0.574	−0.610	0.638	−0.845	0.456	0.857	−0.778	−0.446
14	0.366	0.574	−0.366	0.142	−0.621	0.536	0.271	−0.039	−0.642
15	0.366	0.574	−0.122	−0.254	−0.227	0.604	−0.482	0.156	−0.362
16	0.366	0.191	−0.610	0.638	−0.317	−0.275	0.430	−0.389	0.562
17	0.366	0.191	−0.366	0.057	−0.222	−0.226	0.026	0.368	0.144
18	0.366	0.191	−0.122	−0.389	−0.081	−0.190	−0.271	0.347	−0.056
19	0.366	−0.191	−0.610	0.638	0.317	−0.275	−0.430	−0.389	0.562
20	0.366	−0.191	−0.366	0.057	0.222	−0.226	−0.026	0.368	0.144
21	0.366	−0.191	−0.122	−0.389	0.081	−0.190	0.271	0.347	−0.056
22	0.366	−0.574	−0.610	0.638	0.845	0.456	−0.857	−0.778	−0.446
23	0.366	−0.574	−0.366	0.149	0.621	0.536	−0.271	−0.039	−0.642
24	0.366	−0.574	−0.122	0.254	0.227	0.604	0.482	0.233	−0.362

Table 5 Natural frequencies of the immersed plate with the mass modification

No. of modes	Experimental frequency, Hz	Calculated frequencies, Hz: With mass coupling terms	Calculated frequencies, Hz: Without mass coupling terms
1	46.8	46.8	46.6
2	57.9	58.0	60.5
3	106.2	109.8	114.8
4	127.0	126.9	125.9
5	136.5	137.3	137.3
6	155.9	162.8	166.5

In order to prove the validity of the model, a lumped mass $m = 0.5$ kg was fixed at point P shown in Fig. 1. This mass modification gives new natural frequencies $\overset{*}{\Omega}_i$ and new normal modes $\overset{*}{Y}_i$ which are approximated by the eigenvalue problem,

$$\overset{*}{\Omega}{}_i^2 \left[\begin{bmatrix} I+M_{RR} & M_{RE} \\ M_{ER} & I+M_{EE} \end{bmatrix} + [\Delta M] \right] \overset{*}{Y}_i = \begin{bmatrix} 0 & 0 \\ 0 & \Omega^2 \end{bmatrix} \overset{*}{Y}_i \tag{40}$$

The mass matrix $[\Delta M]$ is given by the relationship,

$$[\Delta M] = m\,[X_R X_E]_p^T\,[X_R X_E]_p \tag{41}$$

The matrix $[X_R X_E]_p$ is composed of the displacements measured at p associated with the normal modes of the "dry" structure. The calculated frequencies $\overset{*}{\Omega}_i$ are very close to those measured experimentally (Table 5). On the other hand, if the mass coupling terms induced by the fluid are discounted, the differences between the calculated frequencies and actual values are greater, as shown in Table 5.

Conclusions

Based on vibration tests, the proposed method gives a discrete model expressing the dynamic behavior of a structure immersed in a fluid. The experimental procedure is based on the measurement of the shape of the first modes of the structure, both "dry" and in contact with the fluid. Applied to the case of a plate partially immersed in water, this method enabled us to predict with accuracy the influence of a modification in the elastic structure. Thus, the discrete model obtained is usable in the context of modal synthesis methods[14-16,18] where linked substructures are not in contact with the fluid.

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